Vektorraum erkunden

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Automatische Übersetzung aus MuPAD 3.11, Mrz 06 Update 14.03.06

Es fehlen nocht textliche Änderungen, die MuPAD 4 direkt berücksichtigen, das ist in Arbeit.

Web: http://haftendorn.uni-lueneburg.de www.mathematik-verstehen.de

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##

1. Vektorraum und seine Gesetze (in 2D)

Vektoren werden als Matrizen aufgefasst.
Eingabe einer "flachen" Liste erzeugt Spaltenvektoren.

```
a := matrix([ax,ay]):
b := matrix([bx,by]):
c := matrix([cx,cy]):a,b,c
```

$$\begin{pmatrix} ax \\ ay \end{pmatrix}, \begin{pmatrix} bx \\ by \end{pmatrix}, \begin{pmatrix} cx \\ cy \end{pmatrix}$$

Nun gelten alle VR-Gesetze

```
a+b
```

$$\begin{pmatrix} ax + bx \\ ay + by \end{pmatrix}$$

```
a+(b+c)=a+(b+c)
```

$$\begin{pmatrix} ax + bx + cx \\ ay + by + cy \end{pmatrix} = \begin{pmatrix} ax + bx + cx \\ ay + by + cy \end{pmatrix}$$

```
ai:=-a
```

$$\begin{pmatrix} -ax \\ -ay \end{pmatrix}$$

```
o:=a+ai
```

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

```
b+o
```

$$\begin{pmatrix} bx \\ by \end{pmatrix}$$

V={a,b,x,o,....} ist mit + eine Gruppe

```
s*a
```

$$\begin{pmatrix} ax \cdot s \\ ay \cdot s \end{pmatrix}$$

```
bool(1*a=a)
```

TRUE

```
Dist1:=(r*(a+b)=r*a+r*b)
```

$$\begin{pmatrix} r \cdot (ax + bx) \\ r \cdot (ay + by) \end{pmatrix} = \begin{pmatrix} ax \cdot r + bx \cdot r \\ ay \cdot r + by \cdot r \end{pmatrix}$$

```
expand(Dist1)
```

$$\begin{pmatrix} ax \cdot r + bx \cdot r \\ ay \cdot r + by \cdot r \end{pmatrix} = \begin{pmatrix} ax \cdot r + bx \cdot r \\ ay \cdot r + by \cdot r \end{pmatrix}$$

```
bool(expand(Dist1))
```

TRUE

```
Dist2:=((r+s)*a=r*a+s*a)
```

$$\begin{pmatrix} ax \cdot (r + s) \\ ay \cdot (r + s) \end{pmatrix} = \begin{pmatrix} ax \cdot r + ax \cdot s \\ ay \cdot r + ay \cdot s \end{pmatrix}$$

```
bool(expand(Dist2))
```

TRUE

```
r*(s*a)=(r*s)*a
```

$$\begin{pmatrix} ax \cdot r \cdot s \\ ay \cdot r \cdot s \end{pmatrix} = \begin{pmatrix} ax \cdot r \cdot s \\ ay \cdot r \cdot s \end{pmatrix}$$

(V,+) bildet mit der Skalaren-Multiplikation einen VR

```
a,b,r,s
```

$$\begin{pmatrix} ax \\ ay \end{pmatrix}, \begin{pmatrix} bx \\ by \end{pmatrix}, r, s$$

###

2. Vektoren 2D visualisieren

```
a := matrix([3,1]);
b := matrix([-2,2]);
```

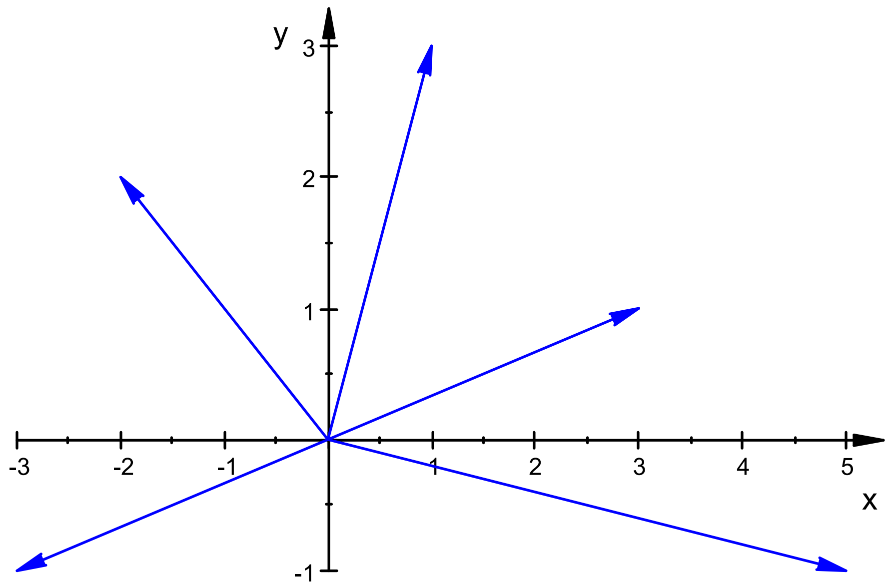
$$\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

```
a,b,a+b,-a,a-b
```

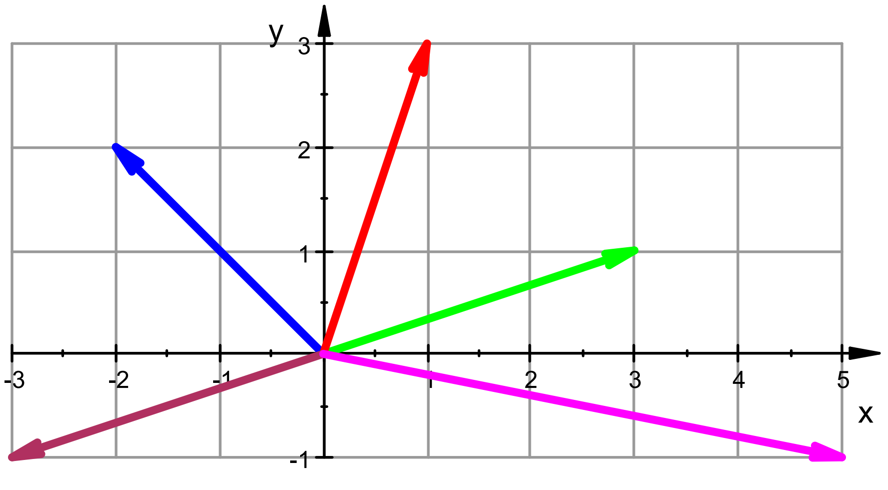
$$\begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{pmatrix} -3 \\ -1 \end{pmatrix}, \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

```
av:=plot::Arrow2d(a):
bv:=plot::Arrow2d(b):
apb:=plot::Arrow2d(a+b):
ai:=plot::Arrow2d(-a):
amb:=plot::Arrow2d(a-b):
plot(av,bv,apb,ai,amb)
```



So kann man nicht erkennen, welches welcher Vektor ist.
(Nette Aufgabe!)

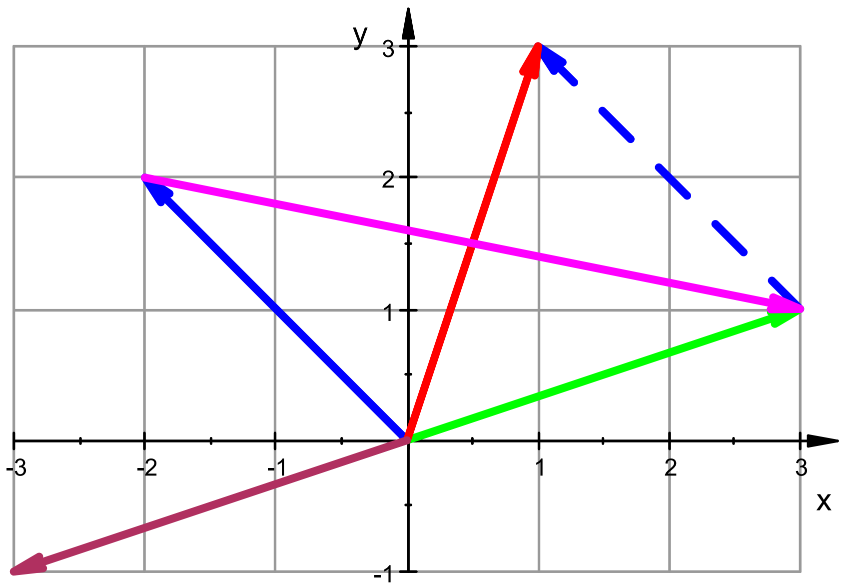
```
av:=plot::Arrow2d(a,LineColor=RGB::Green):
bv:=plot::Arrow2d(b,LineColor=RGB::Blue):
apb:=plot::Arrow2d(a+b,LineColor=RGB::Red):
ai:=plot::Arrow2d(-a,LineColor=RGB::Maroon):
amb:=plot::Arrow2d(a-b,LineColor=RGB::Magenta):
plot(av,bv,apb,ai,amb, LineWidth=1,
                   Scaling=Constrained, GridVisible=TRUE)
```



Für dickere Striche und Karos ist auch noch gesorgt.

Addition als Anhängen, Differen als Spitzenverbindung

```
av:=plot::Arrow2d(a,LineColor=RGB::Green):
bv:=plot::Arrow2d(b,LineColor=RGB::Blue):
bva:=plot::Arrow2d(a,a+b,LineColor=RGB::Blue,LineStyle=Dashe
apb:=plot::Arrow2d(a+b,LineColor=RGB::Red):
ai:=plot::Arrow2d(-a,LineColor=RGB::Maroon):
amb:=plot::Arrow2d(b,a,LineColor=RGB::Magenta):
plot(av,bv,bva,apb,ai,amb, LineWidth=1,
                   Scaling=Constrained, GridVisible=TRUE)
```



Ruft man also der Arrow2D-Befehl mit einem 2. Vektor auf,wird von 4
der Spitze des 1. Vektors zu Spitze des 2. Vektors ein Pfeil gezeichnet.
b an a angehängt realisiert man durch   a,a+b

###
2. Vektoren 3D visualisieren

```
a := matrix([3,1,2]);
b := matrix([-2,2,-1]);
```

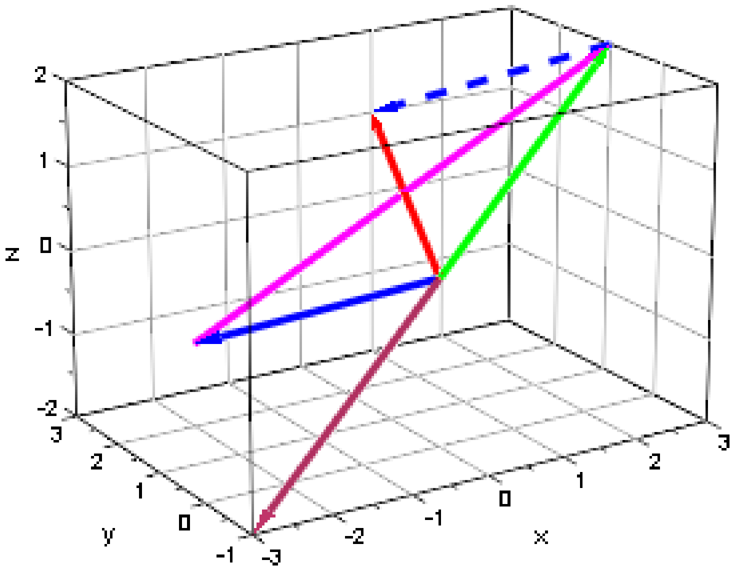
$$\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$$

```
a,b,a+b,-a,a-b
```

$$\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 5 \\ -1 \\ 3 \end{pmatrix}$$

```
av:=plot::Arrow3d(a,LineColor=RGB::Green):
bv:=plot::Arrow3d(b,LineColor=RGB::Blue):
bva:=plot::Arrow3d(a,a+b,LineColor=RGB::Blue,LineStyle=Dashe
apb:=plot::Arrow3d(a+b,LineColor=RGB::Red):
ai:=plot::Arrow3d(-a,LineColor=RGB::Maroon):
amb:=plot::Arrow3d(b,a,LineColor=RGB::Magenta):
plot(av,bv,bva,apb,ai,amb, LineWidth=1,
                    Scaling=Constrained, GridVisible=TRUE)
```



##
3. Vektorraumgesetze 3D visualisieren

```
c := matrix([1,-1,-2]):  a,b,c
```

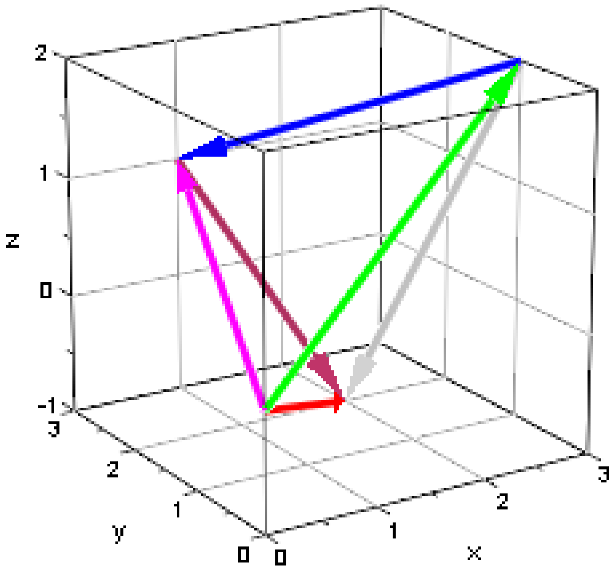
$$\begin{pmatrix}3\\1\\2\end{pmatrix}, \begin{pmatrix}-2\\2\\-1\end{pmatrix}, \begin{pmatrix}1\\-1\\-2\end{pmatrix}$$

Assoziativgesetz

```
a+(b+c)=(a+b)+c
```

$$\begin{pmatrix}2\\2\\-1\end{pmatrix} = \begin{pmatrix}2\\2\\-1\end{pmatrix}$$

```
av:=plot::Arrow3d(a,LineColor=RGB::Green):
bva:=plot::Arrow3d(a,a+b,LineColor=RGB::Blue):
apb:=plot::Arrow3d(a+b,LineColor=RGB::Magenta):
cv:=plot::Arrow3d(a+b,a+b+c,LineColor=RGB::Maroon):
bpc:=plot::Arrow3d(a,a+b+c,LineColor=RGB::Gray):
apbpc:=plot::Arrow3d(a+b+c,LineColor=RGB::Red):
plot(av,bva,apb,cv,bpc,apbpc, LineWidth=1,TipLength=8,
                    Scaling=Constrained, GridVisible=TRUE):
```



Doppelt anklicken, Drehen!!!!
Ursprung am Fuß von grün,rot,magenta

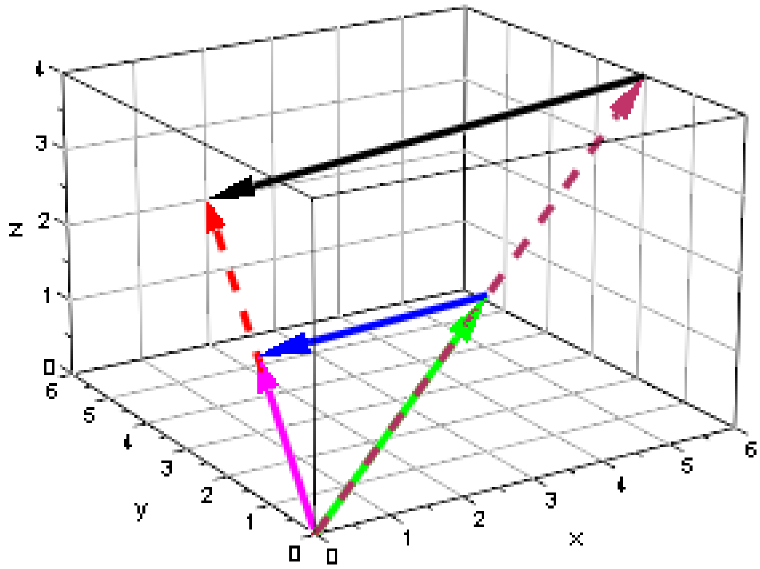
grün+blau+braun=rot
magenta + braun=rot
grün+grau=rot

Distributivgesetz 1 r(a+b)=ra+rb

```
a,b,r*a,r*b,a+b,r*(a+b),r*a+r*b
```

$$\begin{pmatrix}3\\1\\2\end{pmatrix}, \begin{pmatrix}-2\\2\\-1\end{pmatrix}, \begin{pmatrix}3\cdot r\\r\\2\cdot r\end{pmatrix}, \begin{pmatrix}-2\cdot r\\2\cdot r\\-r\end{pmatrix}, \begin{pmatrix}1\\3\\1\end{pmatrix}, \begin{pmatrix}r\\3\cdot r\\r\end{pmatrix}, \begin{pmatrix}r\\3\cdot r\\r\end{pmatrix}$$

```
r:=2:
av:=plot::Arrow3d(a,LineColor=RGB::Green):
bva:=plot::Arrow3d(a,a+b,LineColor=RGB::Blue):
apb:=plot::Arrow3d(a+b,LineColor=RGB::Magenta):
ra:=plot::Arrow3d(r*a,LineColor=RGB::Maroon,LineStyle=Dashed
rbra:=plot::Arrow3d(r*a,r*a+r*b,LineColor=RGB::Black):
rapb:=plot::Arrow3d(r*(a+b),LineColor=RGB::Red,LineStyle=Das
plot(av,bva,apb,ra,rbra,rapb, LineWidth=1,TipLength=8,
                   Scaling=Constrained, GridVisible=TRUE):
```



Doppelt anklicken, Drehen!!!!

Das Distributivgesetz 1 ist der Strahlensatz-Zusammenhang

##

Distributivgesetz 2 $(r+s)\vec{a}=r\vec{a}+s\vec{a}$

```
delete r,s:
a,r,s,(r+s)*a,r*a+s*a,r*a,s*a;
r:=2:s:=3:
av:=plot::Arrow3d(a,LineColor=RGB::Green,LineStyle=Dashed):
rpsa:=plot::Arrow3d((r+s)*a,LineColor=RGB::Magenta):
p1:=plot::Scene3d(av,rpsa,LineWidth=1,TipLength=8,
                   Scaling=Constrained, GridVisible=TRUE):
ra:=plot::Arrow3d(r*a,LineColor=RGB::Maroon,LineStyle=Dashed
sa:=plot::Arrow3d(r*a,r*a+s*a,LineColor=RGB::Black,LineWidth
rapsa:=plot::Arrow3d(r*a+s*a,LineColor=RGB::Red):
p2:=plot::Scene3d(rapsa,ra,sa, LineWidth=1,TipLength=8,
```

```
                                    Scaling=Constrained, GridVisible=TRUE):
plot(p1,p2):
```

$$\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, r, s, \begin{pmatrix} 3\cdot r + 3\cdot s \\ r + s \\ 2\cdot r + 2\cdot s \end{pmatrix}, \begin{pmatrix} 3\cdot r + 3\cdot s \\ r + s \\ 2\cdot r + 2\cdot s \end{pmatrix}, \begin{pmatrix} 3\cdot r \\ r \\ 2\cdot r \end{pmatrix}, \begin{pmatrix} 3\cdot s \\ s \\ 2\cdot s \end{pmatrix}$$