

Null und $1/q$, Funktionenübung

Prof. Dr. Dörte Haftendorn: Mathematik mit MuPAD 4, Sept 07 Update 21.09.07

Web: <http://haftendorn.uni-lueneburg.de>

www.mathematik-verstehen.de

#####

$$f(x) = \begin{cases} 0 & \text{für } x \in \mathbb{R} \setminus \mathbb{Q} \cup \{0,1\} \\ \frac{1}{q} & \text{für } x = \frac{p}{q} \text{ als gekürzter Bruch} \end{cases}$$

1. Unstetigkeitsstellen liegen dicht

2. dennoch Integrierbar

Hairer/Wanner, Analysis by Its History, Springer 1996, S. 224

```
gcd(14, 35)
```

```
7
```

```
fr := (p, q) -> gcd(p, q) / q //rationale Argumente
```

```
(p, q) ->  $\frac{\text{gcd}(p, q)}{q}$ 
```

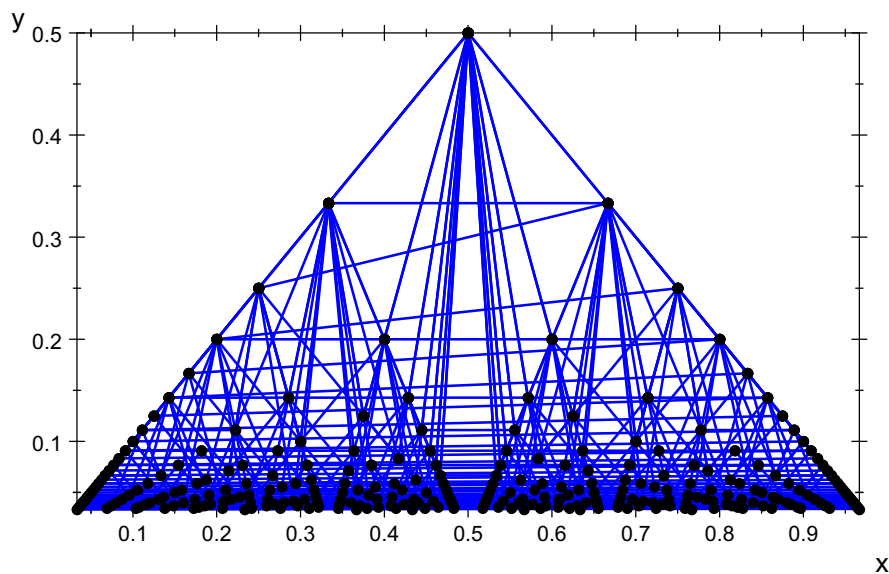
```
([p, q, p/q, fr(p, q)] $ p=1..q-1) $ q=1..4;
```

```
([p, q, p/q, fr(p, q)] $ p=1..q-1) $ q=5..8
```

```
[1, 2,  $\frac{1}{2}$ ,  $\frac{1}{2}$ ], [1, 3,  $\frac{1}{3}$ ,  $\frac{1}{3}$ ], [2, 3,  $\frac{2}{3}$ ,  $\frac{1}{3}$ ], [1, 4,  $\frac{1}{4}$ ,  $\frac{1}{4}$ ], [2, 4,  $\frac{1}{2}$ ,  $\frac{1}{2}$ ], [3, 4,  $\frac{3}{4}$ ,  $\frac{1}{4}$ ]
```

```
[1, 5,  $\frac{1}{5}$ ,  $\frac{1}{5}$ ], [2, 5,  $\frac{2}{5}$ ,  $\frac{1}{5}$ ], [3, 5,  $\frac{3}{5}$ ,  $\frac{1}{5}$ ], [4, 5,  $\frac{4}{5}$ ,  $\frac{1}{5}$ ], [1, 6,  $\frac{1}{6}$ ,  $\frac{1}{6}$ ], [2, 6,  $\frac{1}{3}$ ,  $\frac{1}{3}$ ], [3,
```

```
plot(plot::Listplot([(p/q, fr(p, q)] $ p=1..q-1) $ q=1..30]))
```



1

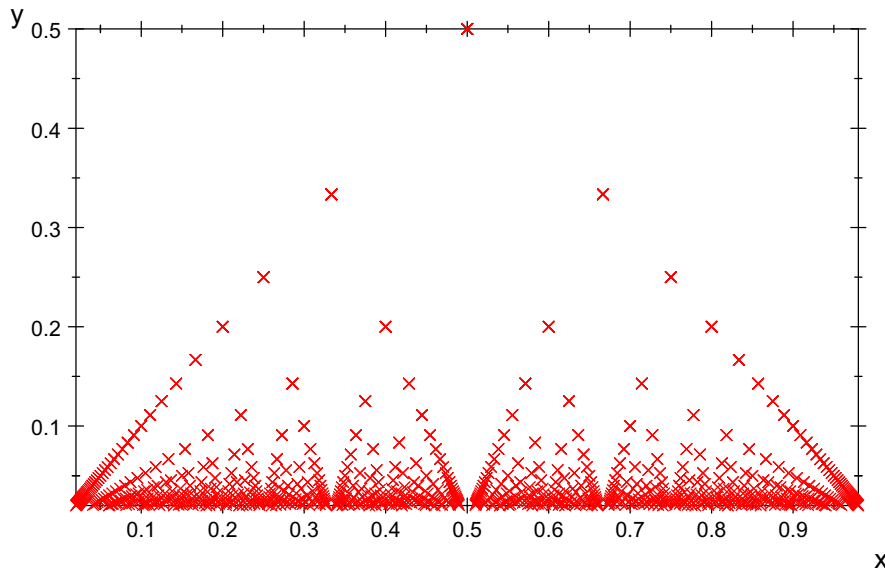
```
fg := plot::Listplot([(p/q, fr(p, q)] $ p=1..q-1) $ q=1..50),
```

```
LinesVisible=FALSE, PointStyle=XCrosses,
```

```

LinesVisible=FALSE, PointStyle=XCrosses,
PointColor=[1,0,0]):
plot (fg)

```



```

n:=6: eps:=1/n; k:=n*(n-1)/2; // oder *56
Dz:=[i/(n*k) $i=1..n*k]:

```

$$\frac{1}{6}$$

15

```

alle:=plot::Rectangle(i/(n*k)..(i+1)/(n*k),0..max(fr(i,
(n*k)),fr((i+1),(n*k)))) $ i=1..n*k-2:

```

```

alle2:=plot::Rectangle(i/(n*k)..
(i+1)/(n*k),0..max(fr(i*56+j,(n*k*56))$ j=0..56 )) $
i=1..n*k-2:

```

```

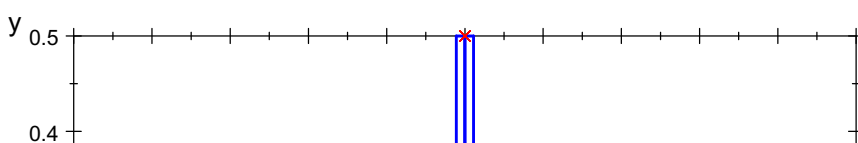
epsg:=plot::Line2d([0,eps],[1,eps],LineColor=[0,0.5,0]):

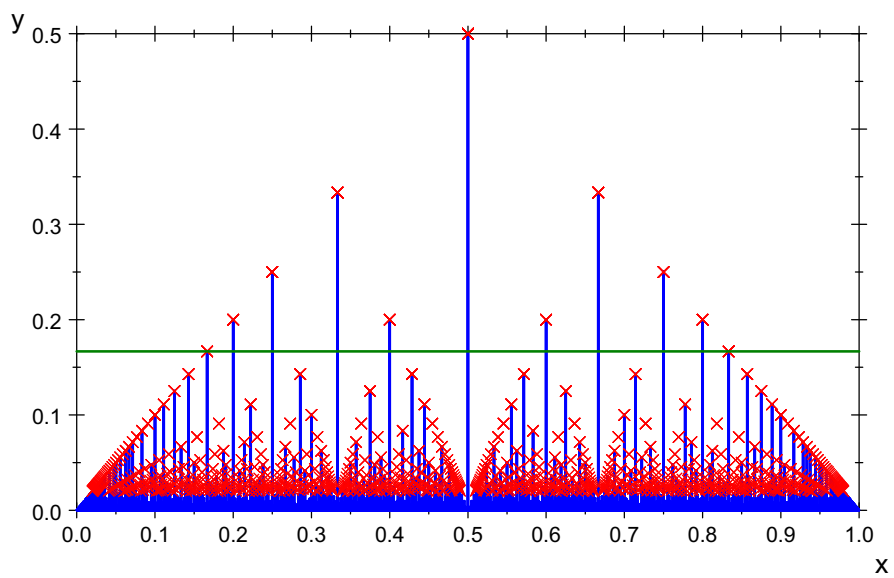
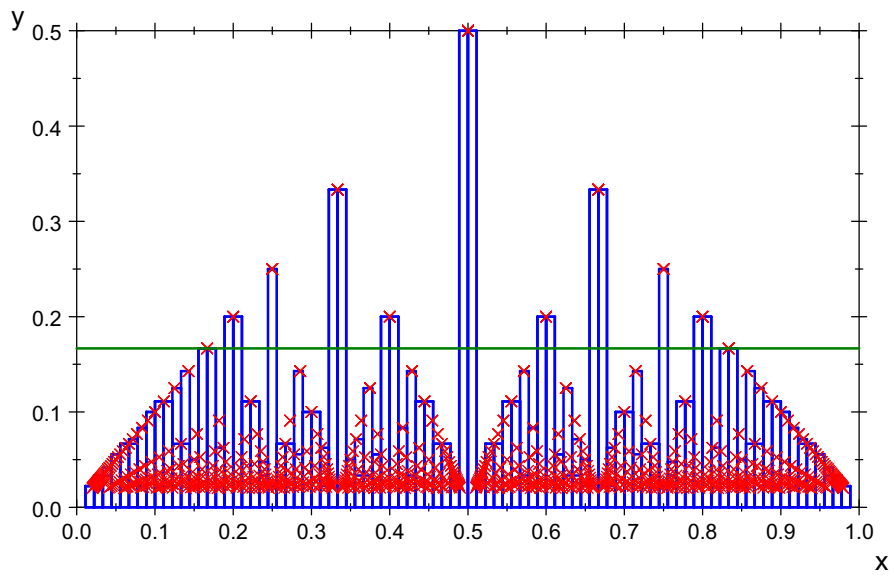
```

```

plot(alle,alle2,fg,epsg)

```





[