DGLn von Hand

$y' - y = \sin x$

Ha 06

homogene $y' - y = 0$

Char. Polynom $k - 1 = 0$

$k = 1$

$y_H = c e^{kx} = c e^x$

spez. Lös Methode Ansatz nach Störfkt.

$y_s = a \sin x + b \cos x$

$y_s' = a \cos x - b \sin x$

$\to a \cos x - b \sin x - (a \sin x + b \cos x) = \sin x$

$(a-b)\cos x + (a+b)\sin x = \sin x$

Wird gelöst von $a - b = 0$

$\wedge\ a + b = 1$

$\oplus$ $2a = 1$

$a = \frac{1}{2}$ $\to b = \frac{1}{2}$

$y_s = \frac{1}{2}\sin x + \frac{1}{2}\cos x$

Allgemeine Lösung $y_{allg} = y_H + y_s = c e^x + \frac{1}{2}\sin x + \frac{1}{2}\cos x$

$y'' - y' - 2y = x^2$

Lösung: Ha 06

homog. $y'' - y' - 2 = 0$

ch. Pol. $k^2 - k - 2 = 0$

Lösen $k^2 - k + \left(\frac{1}{2}\right)^2 = 2 + \left(\frac{1}{2}\right)^2$

$\left(k - \frac{1}{2}\right)^2 = \frac{9}{4}$

$k = \frac{1}{2} \pm \frac{3}{2}$

$k = 2 \vee k = -1$

$y_H = c_1 e^{2x} + c_2 e^{-x}$

$y_s = ax^2 + bx + c$

$y_s' = 2ax + b$

$y_s'' = 2a$

Einsetzen

$2a - (2ax + b) - 2(ax^2 + bx + c) = x^2$

$2a - 2ax - b - 2ax^2 - 2bx - 2c = x^2$

$-2ax^2 - 2(a+b)x + 2a - b - 2c = x^2$

Koeffizientenvergleich

$-2a = 1 \wedge 2(a+b) = 0 \wedge 2a - b - 2c = 0$

$a = -\frac{1}{2}$ $\to b = \frac{1}{2}$ $\to -1 - \frac{1}{2} = 2c$

$2c = -\frac{3}{2}$

$c = -\frac{3}{4}$

$y_s = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{3}{4}$

$y = y_H + y_s = c_1 e^{2x} + c_2 e^{-x} - \frac{1}{2}x^2 + \frac{1}{2}x - \frac{3}{4}$