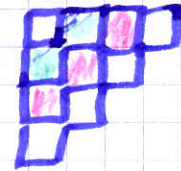


Summe der natürlichen Zahlen



$$\left. \begin{array}{l} 1 \\ 2 \\ 3 \\ \dots \\ 10 \end{array} \right\} = 15$$



Haftendorn
09

$$1 + 2 + 3 + \dots + n = \sum_{i=1}^n i =: S_n$$

$S_1 = 1 \quad S_2 = 3 \quad S_3 = 6 \quad S_4 = 10 \dots$

$$\langle S_n \rangle = \{1, 3, 6, 10, 15, \dots\}$$

Gauß

$$\begin{array}{rcl}
 1 & 2 & 3 & \dots & +99 & +100 & = S_{100} \\
 + & \swarrow & & & & & \\
 + & 100 & +99 & +\dots & & & = S_{100} \\
 \hline
 101 & +101 & +\dots & & & & = 2 \cdot S_{100}
 \end{array}$$

Also

$$\begin{aligned}
 2 \cdot S_{100} &= 100 \cdot 101 \\
 S_{100} &= \frac{100 \cdot 101}{2}
 \end{aligned}$$

analog

$$\begin{array}{rcl}
 1 & +2 & + & & +n-1 & +n & = S_n \\
 + & \swarrow & & & & & \\
 + & n & +n-1 & & & & = S_n \\
 \hline
 n+1 & & & & n+1 & n+1 & = 2S_n
 \end{array}$$

$$2S_n = n \cdot (n+1)$$

$$S_n = \frac{n(n+1)}{2}$$

$$= \sum_{i=1}^n i = \frac{1}{2} n(n+1)$$

Dreieckszahlen, Gaußreihe
 1, 3, 6, 10, ...
 Gaußsumme