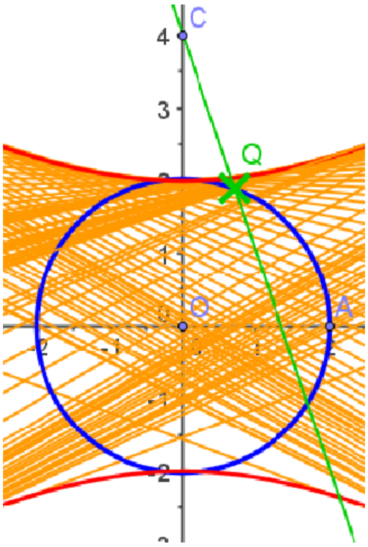
Hyperbel als Hüllkurve

- **Haftendorn Jan 2011 (Aus Reidt-Wolf Bd. 8)**

Konstruktionsbeschreibung :



A sei ein fester Punkt (R, 0) auf der x - Achse. C sei ein fester Punkt (0, c) auf der y - Achse. Q wandert auf dem Kreis um O durch A. Welche Hüllkurve hat die Normalenschar auf CQ in Q?

In[1]:=
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v = Sqrt[R^2 - u^2]
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Out[1]= $\sqrt{R^2 - u^2}$

In[2]:=
```
g[x_, u_] := u / (c - v) (x - u) + v;
g[x, u]
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Out[3]= $\sqrt{R^2 - u^2} + \frac{u\,(-u + x)}{c - \sqrt{R^2 - u^2}}$

In[4]:=
```
g[x, u] // Simplify // Factor
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Out[4]= $\frac{R^2 - c\sqrt{R^2 - u^2} - u\,x}{-c + \sqrt{R^2 - u^2}}$

In[5]:=
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gl = D[g[x, u], u] == 0 // Simplify
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Out[5]= $\frac{-c^2\,u + R^2\,(u - x) + c\sqrt{R^2 - u^2}\;x}{R^2 - u^2 - c\sqrt{R^2 - u^2}} == 0$

In[10]:=
```
lox = Solve[gl, x] // Simplify
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Out[10]= $\left\{\left\{x \to \frac{(c^2 - R^2)\,u}{-R^2 + c\sqrt{R^2 - u^2}}\right\}\right\}$

In[11]:=
```
xx = lox[[1, 1, 2]]
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Out[11]= $\frac{(c^2 - R^2)\,u}{-R^2 + c\sqrt{R^2 - u^2}}$

In[12]:= **yy = g[x, u] /. x → xx // Factor**

Out[12]= $-\frac{R^2\left(c-\sqrt{R^2-u^2}\right)}{R^2-c\sqrt{R^2-u^2}}$

In[13]:= **xx**

Out[13]= $\frac{\left(c^2-R^2\right)u}{-R^2+c\sqrt{R^2-u^2}}$

In[14]:= **{xx, yy}**

Out[14]= $\left\{\frac{\left(c^2-R^2\right)u}{-R^2+c\sqrt{R^2-u^2}},\ -\frac{R^2\left(c-\sqrt{R^2-u^2}\right)}{R^2-c\sqrt{R^2-u^2}}\right\}$

In[41]:= **hyp = Eliminate[{x == xx, y == yy}, {u}]**

Out[41]= $-c^2 R^2+R^4+c^2 y^2-R^2 y^2 == R^2 x^2$

In[42]:= **hyp /. {R → 2, c → 4}**

Out[42]= $-48+12 y^2 == 4 x^2$

U-Auflösung (klappt nicht)

In[30]:= **gl**

Out[30]= $\frac{-c^2 u+R^2 (u-x)+c\sqrt{R^2-u^2}\ x}{R^2-u^2-c\sqrt{R^2-u^2}} == 0$

In[32]:= **gl[[1]]**

Out[32]= $\frac{-c^2 u+R^2 (u-x)+c\sqrt{R^2-u^2}\ x}{R^2-u^2-c\sqrt{R^2-u^2}}$

In[33]:= **glu = Numerator[gl[[1]]] == 0**

Out[33]= $-c^2 u+R^2 (u-x)+c\sqrt{R^2-u^2}\ x == 0$

In[40]:= **lo = Solve[glu, u]**

Out[40]= $\left\{\left\{u\to\left(-c^2 R^2 x+R^4 x-\sqrt{\left(c^6 R^2 x^2-2 c^4 R^4 x^2+c^2 R^6 x^2+c^4 R^2 x^4-c^2 R^4 x^4\right)}\right)\Big/\left(c^4-2 c^2 R^2+R^4+c^2 x^2\right)\right\},\right.$
$\left.\left\{u\to\left(-c^2 R^2 x+R^4 x+\sqrt{\left(c^6 R^2 x^2-2 c^4 R^4 x^2+c^2 R^6 x^2+c^4 R^2 x^4-c^2 R^4 x^4\right)}\right)\Big/\left(c^4-2 c^2 R^2+R^4+c^2 x^2\right)\right\}\right\}$

In[35]:= **lo[[2, 1]]**

Out[35]= $u\to\left(-c^2 R^2 x+R^4 x+\sqrt{\left(c^6 R^2 x^2-2 c^4 R^4 x^2+c^2 R^6 x^2+c^4 R^2 x^4-c^2 R^4 x^4\right)}\right)\Big/\left(c^4-2 c^2 R^2+R^4+c^2 x^2\right)$