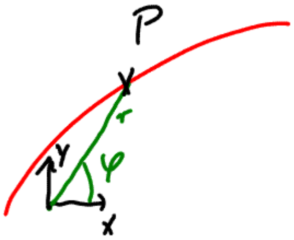
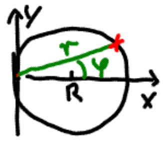
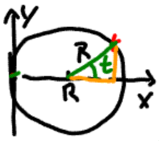
Parameterdarstellung vs Polardarstellung



$P=(x,y)$	$P=(x,y)$	$P=(r,\varphi)$
$x=x(t)$	$x=r(\varphi)\cos\varphi$	$r^2=x^2+y^2$
$y=y(t)$	$y=r(\varphi)\sin\varphi$	$r=r(\varphi)$





$(x-R)^2+y^2=R^2 \Rightarrow (r\cos\varphi-R)^2+r^2\sin^2\varphi=R^2$

$r^2(\cos^2\varphi+\sin^2\varphi)-2rR\cos\varphi+R^2=R^2$

$r^2-2rR\cos\varphi=0$

$r(r-2R\cos\varphi)=0$

Polargleichung $r=2R\cos\varphi$

aus der impliziten Gl.

Parameterdarstellung

$x=R+R\cos t$

$y=R\sin t$

Polargl. aus der Parameterdarstellung

$r^2=x^2+y^2=R^2\left((1+\cos t)^2+\sin^2 t\right)$

$r^2=R^2(1+2\cos t+1)=2R^2(1+\cos t)$

Es ist $t=2\varphi$

$\cos 2\varphi=\cos^2\varphi-\sin^2\varphi$

$=2R^2(1+\cos^2\varphi-\sin^2\varphi)$

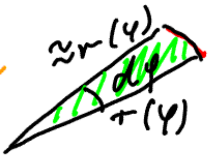
$r^2=2R^2\cdot 2\cos^2\varphi=4R^2\cos^2\varphi$

Polargleichung $r=2R\cos\varphi$

Ableitung $y=f(x)$ $\quad f'(x)=\frac{dy}{dx}=\frac{dy(t)}{dt}\cdot\frac{dt}{dx(t)}=\frac{\dot y}{\dot x}$

Polar $\quad x=r(t)\cos t,\ y=r(t)\sin t \quad \dot x=\dot r\cos t-r\sin t,\ \dot y=\dot r\sin t+r\cos t \quad \frac{dy}{dx}=\frac{\dot r\sin t+r\cos t}{\dot r\cos t-r\sin t}$

Flächen Polar

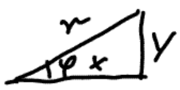


$\frac{\text{Sektor}}{\pi r(\varphi)^2}=\frac{d\varphi}{2\pi} \Longrightarrow \text{Sektor}=\frac{1}{2}r(\varphi)^2\cdot d\varphi$

$A=\frac{1}{2}\int_{\varphi_0}^{\varphi_1} r^2\,d\varphi$

Flächen aus Parameterdarstellung

Arens S. 861 f $\quad A=\frac{1}{2}\int_{\varphi_0}^{\varphi_1}\left(x(t)^2+y(t^2)\right)\cdot\frac{d\varphi}{dt}\cdot dt$



$\varphi=\arctan\frac{y}{x} \Rightarrow \frac{d\varphi}{dt}=\frac{1}{1+\frac{y^2}{x^2}}\cdot\frac{\dot y x-y\dot x}{x^2}\,dt=\frac{x^2(\dot y x-y\dot x)}{(x^2+y^2)x^2}\,dt$

Also $A=\frac{1}{2}\int_{\varphi_0}^{\varphi_1}(\dot y x-y\dot x)\,dt$ $\quad$ mit t_0 aus $x(t_0)=r(\varphi_0)\cdot\cos\varphi_0$

t_1 entspr. $\quad y(t_0)=r(\varphi_0)\sin\varphi_0$

$$A=\frac{1}{2}\int_{t_0}^{t_1}(\dot y x-y\dot x)\,dt$$

Bogenlänge $\quad s=\int_{x_0}^{x_1}\sqrt{1+f'(x)^2}\,dx=\int_{t_0}^{t_1}\sqrt{\dot x^2+\dot y^2}\,dt=\int_{\varphi_0}^{\varphi_1}\sqrt{r^2+\dot r^2}\,d\varphi$