logarithmische Spirale, durch Kreisbögen angenähert

siehe dazu eine Geogebra-Datei: logarithmischeSpirale-mitKreisen.ggb

- **der goldene Schnitt**

- **Punkt F und Punkt G (Zentrum der Spirale), Parameter der Spirale**

$$\textbf{solG} = \textbf{Solve}\left[\left\{\frac{\textbf{a}}{\textbf{b}} == \phi - 1,\ \frac{\textbf{b}}{1-\textbf{a}} == \phi - 1\right\}\right][[1]]\ //\ \textbf{Simplify}$$

$$\left\{a \to \frac{1}{10}\left(5-\sqrt{5}\right),\ b \to \frac{1}{\sqrt{5}}\right\}$$

$$\textbf{F} = \{\textbf{b} - \phi + 1,\ 1 - \textbf{a}\}\ /.\ \textbf{solG}\ //\ \textbf{Simplify}$$

$$\left\{\frac{1}{10}\left(5-3\sqrt{5}\right),\ \frac{1}{10}\left(5+\sqrt{5}\right)\right\}$$

F // N

{-0.17082, 0.723607}

$$\textbf{x}[\alpha\_] := \textbf{s}\,\phi^{\frac{\alpha}{\pi/2}}\,\textbf{Cos}[\alpha]$$

$$\textbf{y}[\alpha\_] := \textbf{s}\,\phi^{\frac{\alpha}{\pi/2}}\,\textbf{Sin}[\alpha]$$

parms = FindRoot[{x[α] == F[[1]], y[α] == F[[2]]}, {{α, π / 2}, {s, 1}}]

{α → 1.80262, s → 0.428004}

α / (1 °) /. parms

103.283

- **Rumspielen mit algebraischen Zahlen: wo genau ist die Spiralmitte G**

woa = ToNumberField[a /. solG, ϕ]

$$\text{AlgebraicNumber}\left[\frac{1}{2}\left(1+\sqrt{5}\right),\ \left\{\frac{3}{5},\ -\frac{1}{5}\right\}\right]$$

wob = ToNumberField[b /. solG, ϕ]

$$\text{AlgebraicNumber}\left[\frac{1}{2}\left(1+\sqrt{5}\right),\ \left\{-\frac{1}{5},\ \frac{2}{5}\right\}\right]$$

woa[[2]].{1, ϕ // HoldForm}

$$\frac{3}{5} - \frac{\phi}{5}$$

wob[[2]].{1, ϕ // HoldForm}

$$-\frac{1}{5} + \frac{2\phi}{5}$$

- **Ein weiterer Punkt auf der Spirale ist T, der Schnitt des Kreises im Quadrat ABEF mit der Diagonalen g im Rechteck ABCD**

```
B = {-ϕ + b, -a} /. solG // Simplify
```

$$\left\{\frac{1}{10}\left(-5-3\sqrt{5}\right),\ \frac{1}{10}\left(-5+\sqrt{5}\right)\right\}$$

```
lf = F.F // Simplify
```

$$1-\frac{1}{\sqrt{5}}$$

```
lb = B.B // Simplify
```

$$1+\frac{1}{\sqrt{5}}$$

```
Ep = {b - ϕ + 1, -a} /. solG // Simplify
```

$$\left\{\frac{1}{10}\left(5-3\sqrt{5}\right),\ \frac{1}{10}\left(-5+\sqrt{5}\right)\right\}$$

```
g[x_] := -(ϕ - 1) x
```

```
solT = Solve[Norm[{x, g[x]} - Ep] == 1, x][[1]] // Simplify
```

$$\left\{x\to-\sqrt{\frac{1}{5}+\frac{1}{\sqrt{5}}}\right\}$$

```
T = {x, g[x]} /. solT // Simplify
```

$$\left\{-\sqrt{\frac{1}{5}+\frac{1}{\sqrt{5}}},\ \frac{1}{2}\sqrt{\frac{1}{5}+\frac{1}{\sqrt{5}}}\left(-1+\sqrt{5}\right)\right\}$$

- $|T|^2 = |F|\ |B|$

```
lt = T.T // Simplify
```

$$\frac{2}{\sqrt{5}}$$

```
lt^2 - lb lf // Simplify
```

0

- **Die Gerade g halbiert den Winkel FGB**

$$\frac{\mathbf{T.F}}{\sqrt{\mathbf{T.T}}\ \sqrt{\mathbf{F.F}}}\ \texttt{// Simplify}$$

$$\frac{1}{\sqrt{2}}$$

Die logarithmische Spirale

Vorbereitung

```
X[α_] := x[α] /. parms[[2]] // Evaluate
```

```
Y[α_] := y[α] /. parms[[2]] // Evaluate
```

```
Y[α]
```

$$0.428004 \left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{\frac{2\alpha}{\pi}} \mathrm{Sin}[\alpha]$$

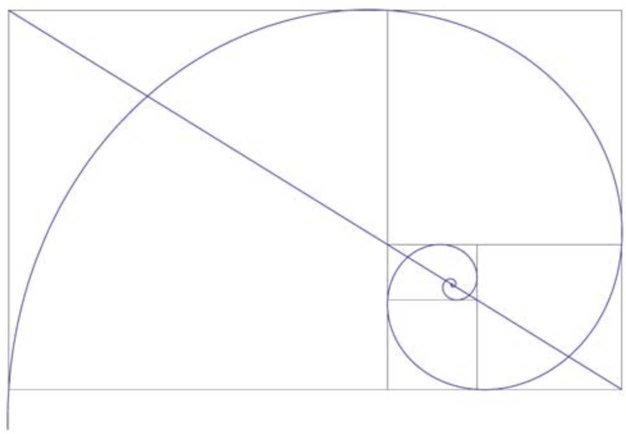
```
geradeg = Plot[g[x], {x, b - ϕ, b} /. solG // Evaluate];
```

```
curve = ParametricPlot[{X[α], Y[α]}, {α, -10 π, 1.1 π}, PlotRange → All];
```

```
quads =
  Graphics[{EdgeForm[Thin], Opacity[0], Rectangle[{b - ϕ, -a}, {b - ϕ + 1, -a + 1}], Rectangle[
      {b - ϕ + 1, -a + 1 + 1 - ϕ}, {b, -a + 1}], Rectangle[{b - 2 + ϕ, -a}, {b, -a + 1 + 1 - ϕ}],
     Rectangle[{b - ϕ + 1, -a}, {b - 2 + ϕ, 2 ϕ - a - 3}]}] /. solG;
```

Bild

```
Show[quads, curve, geradeg]
```



Die Spirale passt nicht ganz in die Quadrate

```
top = FindRoot[Y'[α] == 0, {α, π / 2}]
```

$\{\alpha \to 1.86807\}$

```
t1 = Y[α] /. top
```

0.725283

```
t1 / t2
```

$$\frac{0.725283}{t2}$$

```
t2 = X[α] /. top
```

-0.22219

```
Show[quads, curve,
 PlotRange → {{t2 - 0.1, F[[1]] + 0.03}, {F[[2]] - 0.03, t1 + 0.01}}, AspectRatio → Automatic]
```